

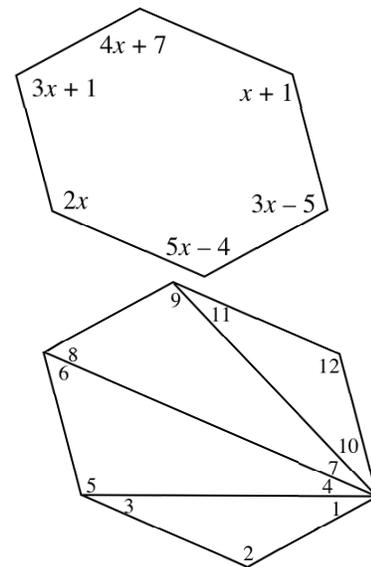
After studying triangles and quadrilaterals, students now extend their study to all polygons. A polygon is a closed, two-dimensional figure made of three or more non-intersecting straight line segments connected end-to-end. Using the fact that the sum of the measures of the angles in a triangle is 180° , students learn a method to determine the sum of the measures of the interior angles of any polygon. Next they explore the sum of the measures of the exterior angles of a polygon. Finally they use the information about the angles of polygons along with their Triangle Toolkits to find the areas of regular polygons.

See the Math Notes boxes in Lessons 8.1.1, 8.1.5, and 8.3.1.

Example 1

The figure at right is a hexagon. What is the sum of the measures of the interior angles of a hexagon? Explain how you know. Then write an equation and solve for x .

One way to find the sum of the interior angles of the hexagon is to divide the figure into triangles. There are several different ways to do this, but keep in mind that we are trying to add the interior angles at the vertices. One way to divide the hexagon into triangles is to draw in all of the diagonals from a single vertex, as shown at right. Doing this forms four triangles, each with angle measures summing to 180° .



$$\underbrace{m\angle 1 + m\angle 2 + m\angle 3}_{180^\circ} + \underbrace{m\angle 4 + m\angle 5 + m\angle 6}_{180^\circ} + \underbrace{m\angle 7 + m\angle 8 + m\angle 9}_{180^\circ} + \underbrace{m\angle 10 + m\angle 11 + m\angle 12}_{180^\circ} = 4(180^\circ) = 720^\circ$$

(Note: Students may have noticed that the number of triangles is always two less than the number of sides. This example illustrates why the sum of the interior angles of a polygon may be calculated using the formula $(n - 2)180^\circ$, where n is the number of sides of the polygon.)

Now that we know what the sum of the angles is, we can write an equation, and solve for x .

$$\begin{aligned} (3x + 1) + (4x + 7) + (x + 1) + (3x - 5) + (5x - 4) + (2x) &= 720^\circ \\ 18x &= 720^\circ \\ x &= 40^\circ \end{aligned}$$

Example 2

If the sum of the measures of the interior angles of a polygon is 2340° , how many sides does the polygon have?

Use the equation “sum of interior angles = $(n - 2)180^\circ$ ” to write an equation and solve for n . The solution is shown at right.

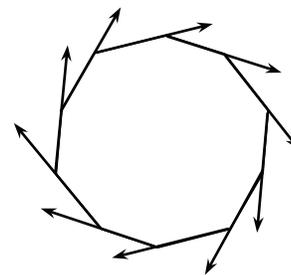
$$\begin{aligned}(n - 2) \cdot 180^\circ &= 2340^\circ \\ 180n - 360 &= 2340 \\ 180n &= 2700 \\ n &= 15\end{aligned}$$

Since $n = 15$, the polygon has 15 sides. It is important to note that if the answer is not a whole number, an error was made or there is no polygon with its interior angles summing to the measure given. Since the answer is the number of sides, the answer can only be a whole number. Polygons cannot have “7.2” sides!

Example 3

What is the measure of an exterior angle of a regular decagon?

A decagon is a 10-sided polygon. Since this figure is a regular decagon, all the angles and all the sides are congruent. The sum of the measures of the exterior angles of any polygon, one at each vertex, is always 360° , no matter how many sides the polygon has. In this case the exterior angles are congruent since the decagon is regular. The decagon at right has ten exterior angles drawn, one at each vertex. Therefore, each angle measures $\frac{360^\circ}{10} = 36^\circ$.

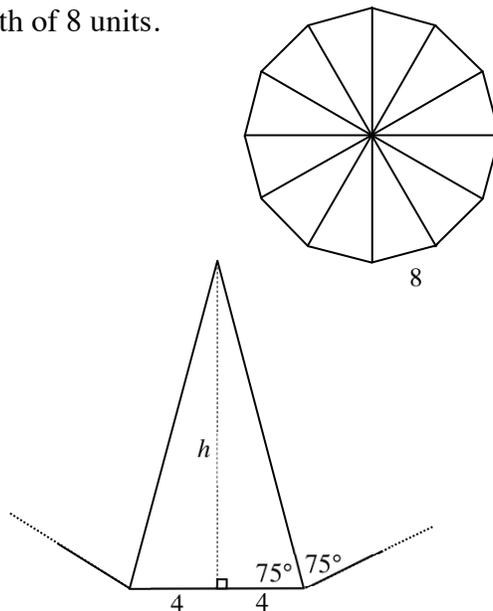


Example 4

A regular dodecagon (12 sided polygon) has a side length of 8 units. What is its area?

Solving this problem is going to require the use of several topics that have been studied. (Note: There is more than one way to solve this problem.) For this solution, we will imagine dividing the dodecagon into 12 congruent triangles, radiating from the center. If we find the area of one of them, then we can multiply it by 12 to get the area of the entire figure.

To focus on one triangle, copy and enlarge it. The triangle is isosceles, so drawing a segment from the vertex perpendicular to the base gives a height. This height also bisects the base (because this triangle is isosceles).



Since this is a dodecagon, we can find the sum of all the angles of the shape by using the formula:

$$(12 - 2)(180^\circ) = 1800^\circ$$

Since all the angles are congruent, each angle measures $1800^\circ \div 12 = 150^\circ$. The segments radiating from the center bisect each angle, so the base angle of the isosceles triangle is 75° . Now we can use trigonometry to find h .

$$\begin{aligned} \tan 75^\circ &= \frac{h}{4} \\ h &= 4 \tan 75^\circ \\ h &\approx 14.928 \end{aligned}$$

Therefore the area of one of these triangles is: $A \approx \frac{1}{2}(8)(14.928) \approx 59.712$ square units

To find the area of the dodecagon, we multiply the area of one triangle by 12.

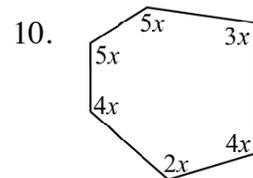
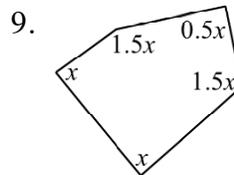
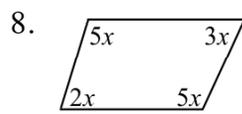
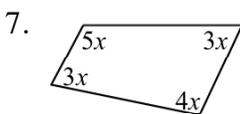
$$A \approx 12(59.712) \approx 716.544 \text{ square units}$$

Problems

Find the measures of the angles in each problem below.

- Find the sum of the interior angles in a 7-gon.
- Find the sum of the interior angles in an 8-gon.
- Find the size of each of the interior angle of a regular 12-gon.
- Find the size of each of the interior angle of a regular 15-gon.
- Find the size of each of the exterior angle of a regular 17-gon.
- Find the size of each of the exterior angle of a regular 21-gon.

Solve for x in each of the figures below.

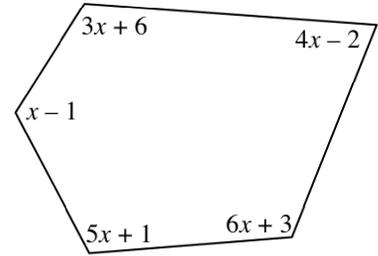


Complete each of the following problems.

- Each exterior angle of a regular n -gon measures $16\frac{4}{11}^\circ$. How many sides does this n -gon have?
- Each exterior angle of a regular n -gon measures $13\frac{1}{3}^\circ$. How many sides does this n -gon have?
- Each angle of a regular n -gon measures 156° . How many sides does this n -gon have?
- Each angle of a regular n -gon measures 165.6° . How many sides does this n -gon have?
- Find the area of a regular pentagon with side length 8 cm.
- Find the area of a regular hexagon with side length 10 ft.
- Find the area of a regular octagon with side length 12 m.

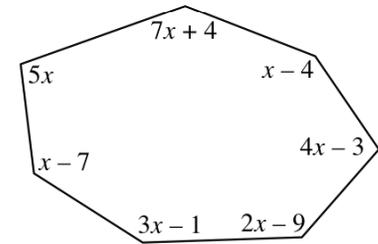
18. Find the area of a regular decagon with side length 14 in.

19. Using the pentagon at right, write an equation and solve for x .



20. Using the heptagon (7-gon) at right, write an equation and solve for x .

21. What is the sum of the measures of the interior angles of a 14-sided polygon?



22. What is the measure of each interior angle of a regular 16-sided polygon?

23. What is the sum of the measures of the exterior angles of a decagon (10-gon)?

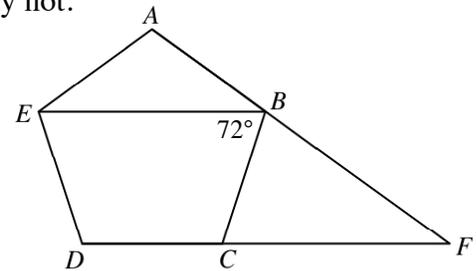
24. Each exterior angle of a regular polygon measures 22.5° . How many sides does the polygon have?

25. Does a polygon exist whose sum of the interior angles is 3060° ? If so, how many sides does it have? If not, explain why not.

26. Does a polygon exist whose sum of the interior angles is 1350° ? If so, how many sides does it have? If not, explain why not.

27. Does a polygon exist whose sum of the interior angles is 4410° ? If so, how many sides does it have? If not, explain why not.

28. In the figure at right, $ABCDE$ is a regular pentagon. Is $\overline{EB} \parallel \overline{DF}$? Justify your answer.



29. What is the area of a regular pentagon with a side length of 10 units?

30. What is the area of a regular 15-gon with a side length of 5 units?

Answers

1. 900°
2. 1080°
3. 150°
4. 156°
5. 21.1765°
6. 17.1429°
7. $x = 24^\circ$
8. $x = 30^\circ$
9. $x = 98.18^\circ$
10. $x = 31.30^\circ$
11. 22 sides
12. 27 sides
13. 15 sides
14. 25 sides
15. 110.1106 cm^2
16. 259.8076 ft^2
17. 695.2935 m^2
18. 1508.0649 in.^2
19. $19x + 7 = 540, x \approx 28.05$
20. $23x - 20 = 900, x = 40$
21. 2160°
22. 157.5°
23. 360°
24. 16 sides
25. 19 sides
26. No. The result is not a whole number.
27. No. The result is not a whole number.
29. Yes. Since $ABCDE$ is a regular pentagon, the measure of each interior angle is 108° . Therefore, $m\angle DCB = 108^\circ$. Since $\angle DCB$ and $\angle FCB$ are supplementary, $m\angle FCB = 72^\circ$. The lines are parallel because the alternate interior angles are congruent.
30. ≈ 172.0 sq. units
31. ≈ 441.1 sq. units